

BOOK REVIEW

Turbulence. By P. A. DAVIDSON. Oxford University Press, 2004. 678 pp. ISBN 0 19 852948 1, £95.00 (hardback) or ISBN 0 19 852949 X, £35.00 (paperback)
J. Fluid Mech. (2005), vol. 540, doi:10.1017/S0022112005005902

The studying of turbulence involves first understanding many aspects of applied mathematics, probability and statistics, statistical physics, as well as the fundamentals of fluid mechanics for inviscid and viscous flows. Then one has to study how these elements are brought together for the discussion and analysis of the various aspects of turbulence. No one person or textbook can cover every aspect. The main methodological question for authors of textbooks is to decide how much to explain the relevant mathematical, physical and fluid mechanics fundamentals, and whether to explain them in isolation or to explain them as one proceeds with the analysis of the various aspects of turbulent phenomena, and the approximate statistical models used to describe them. The popular textbook by Tennekes & Lumley (1972, reviewed in *JFM*, vol. 58, 1973) plunges into a highly physical discussion of the dissipation of turbulent energy in chapter 1, like a good detective story that always begins with the crime. This certainly engages the reader, but for a student trained in the fundamentals of fluid mechanics perhaps too many new special features of turbulence are introduced too early. That book written in the early 1970s has not been brought up to date to include more recent developments of turbulence research. There have been more recent introductory textbooks. Mathieu & Scott (2000), reviewed in *JFM*, vol. 429, 2001, describes the physical concepts of turbulence and some applications but these are not related in detail to the basic equations and fluid mechanics theory. The textbooks by Pope (2000) and Durbin & Petterson-Reif (2001), reviewed in *JFM*, in vols. 427 and 447, 2001) focus on statistical models of inhomogeneous turbulence, including some of the latest developments. Both have substantial sections reviewing the conventional ideas about multi-scale dynamics of homogeneous turbulence. Tsinober's (2002) monograph, reviewed in *JFM*, vol. 475, 2003) reviews new ideas about the cascade dynamics and also the special features of inhomogeneous turbulence at the outer edges of turbulent flows.

Davidson's book is aimed at students who want to know about the main features of turbulent flows, but also how these relate to the fundamental physical and mathematical questions about the mechanisms of turbulence. The style of the book is discursive (500+ pages) and diagrammatic, combining physical and deterministic analyses of basic fluid mechanics mechanisms, e.g. vortex stretching, with introducing stochastic aspects both locally and globally. The mathematics is brought in as needed. It starts in chapters 2 from the equation of motion concepts of vorticity dynamics and kinematical theory and statistical theory, and shows broadly how these are used to describe and analyse turbulent flows.

Chapter 3 briefly introduces the instability of laminar flow and then focuses on bifurcation as a route to chaos, and transition to turbulence. Davidson draws the very reasonable physical analogy with the logistic equation to explain chaos. But this misses some aspects of the Ruelle–Takens–Gollub–Swinney theoretical and experimental

discovery in the 1970s of how a continuous spectrum forms at finite Reynolds number – in contradiction to Landau's (1944) conjecture about a growing sequence of line spectra. Another qualitative development.

A physical explanation for the very long memory of the large-scale eddies in turbulent flows – which is an intriguing and essential phenomenon of turbulence that distinguishes it from the molecular behaviour of gases – is given in terms of the angular (or other kinds) of momentum of random motions integrated over volumes. But studies of various types of turbulent flows have shown how the memory is also determined by particular processes such as merging or splitting of large eddies and how they are affected by initial conditions. One of the reasons for the sensitivity to initial conditions of large eddies is that their growth rate is algebraic and not exponential, an important conceptual development of modern turbulence theory.

In Chapter 4 on turbulent shear flows the traditional statistical models are explained well for the mean velocity profiles for boundary layers, jets and wakes. The more detailed knowledge of coherent structures (at least for low Reynolds numbers) is reviewed. Although it is mentioned that they affect the mean flow profiles and the statistics of the fluctuations, the connections are not quantified. These structures probably determine the remarkable sensitivity of turbulent shear flows to certain kinds of disturbances in particular conditions, such as the effects of weak pressure gradients or stratification on boundary layers (first pointed by Bradshaw) or the variation of the slope of a jet from circular to elliptical (e.g. Husain & Hussain 1991) is yet another example of the great difference between turbulent eddies and gas molecules. As O. M. Philips and A. A. Townsend pointed out there is also another kind of structural difference; sheared turbulence is in some respects similar to wave motion or to the behaviour of certain kinds of elastic media – an important property when turbulence undergoes rapid distortion.

Chapters 5 to 8 on the dynamics of homogeneous turbulence are the heart of the book and the most original contribution. The results of experiments and numerical simulations of turbulence provide details of the large- and small-scale eddy motion, and the mechanisms for transferring energy to the small scales and hence dissipating it (discovered by G. I. Taylor in 1938, although intuited earlier by L. F. Richardson).

The vortex tubes and sheets involved in this process can be understood in terms of deterministic fluid mechanics. But, as Landau foresaw, without detailed measurements it is difficult to relate these highly intermittent mechanisms to the statistical physics theory of Kolomogorov (1941) and Obukhov (1941) based on the concept of the mean rate of dissipation. Davidson does not duck these awkward issues, but provides extensive comparisons of the competing mathematical models for these intermittent processes in small-scale turbulence. It is likely that the nature of the eddy structures over varying length scales might explain a fundamental question now being asked about the Kolmogorov–Obukhov theory, namely to what extent is turbulent eddy structure statistically self-similar, and over what range of scales. The answer varies greatly according to which statistical measure is used (spectra, correlation, fractal dimension, etc. Davidson does not like 'fractals'; there is no mention in the appendix, though space filling is covered!).

The growth of large-scale eddies also drains energy from the energy-containing eddies. Understanding their structure and dynamics is therefore equally important for predicting the evolution of the spectrum of turbulence as it decays, or as it responds to energy from external sources. Chapter 6 shows clearly how the integrals of linear momentum and angular momentum of these large-scale motions broadly determines how their energy (e.g. as defined by the spectrum the spectrum) varies with their

length scales, and how the length scales increase through continual interactions and merging of eddies. It is pointed out that there are uncertainties (or chaotic properties?) associated with the initial conditions which greatly affect these integrals. Perhaps this relates to delicate mathematical questions about the conditionally convergent integrals for the energy and impulse of moving eddies (similar issues arise when calculating these quantities for moving bodies (Eames, Belcher & Hunt 1994). The shape of the limiting infinite volume has to be chosen carefully.

I agree with Davidson's conclusion that for most turbulent flows $E(k)\propto k^4$ as $k \rightarrow 0$ (which is consistent with the form of irrotational fluctuations outside turbulent shear layers). Numerical simulations by Professor Kaneda using the Japanese Earth Simulator may provide a definitive answer before long.

Part III of the book describes the special topics of the effects on homogeneous turbulence of body forms caused by buoyancy forces, rotation and electromagnetism (or magnetohydrodynamics), the subject of most of Davidson's own research. Excellent diagrams and geophysical examples illustrate the mechanisms and their wider significance. The effects on individual vortical eddies are analysed using physical argument and order-of-magnitude equations. These techniques are used to derive results in a general form for the decay of turbulence under the influence of a uniform magnetic field. When the body forces are strong enough some of these results can be deduced by linear theory (cf. Moffatt 1967). For modelling and for understanding it is important to know which effects are linear and which are nonlinear. (For example in the presence of fixed boundaries, body forces lead to very sensitive nonlinear effects.)

I enjoyed reading this book, including the many amusing and apposite quotations. As a turbulence researcher I was stimulated by the clear presentation of critical problems that we ought to understand better. It is not surprising that I have heard good reports from colleagues who are using this text in conjunction with their turbulence courses for final year/postgraduate engineering students. Engineers, as well as physicists, need to use a book like this to improve their understanding of the basic concepts of turbulence in order that their use of numerical models does not lead them astray. I can strongly recommend it to readers of *JFM*.

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